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Energetics of Forced Thermal Ratchet

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Molecular motors are known to have the high efficiency of energy transformation in the presence of thermal fluctuation[1]. Motivated by the surprising fact, recent studies of thermal ratchet models[2] are showing how and when work should be extracted from non-equilibrium fluctuations[3, 4, 5, 6, 7, 8, 9, 10]. One of the important finding was brought by Magnasco[6] where he studied the temperature dependence on the fluctuation-induced current in a ratchet (multistable) system and showed that the current can generically be maximized in a finite temperature. The interesting finding has been interpreted that thermal fluctuation is *not* harmful for the fluctuation-induced work and even facilitates its efficiency. We show, however, this interpretation turns out to be incorrect as soon as we go into the realm of the energetics[11]: the efficiency of energy transformation is not maximized at finite temperature, even in the same system[6] that Magnasco considered. The maximum efficiency is realized in the absence of thermal fluctuation. The result presents an open problem whether thermal fluctuation could facilitate the efficiency of energetic transformation from force-fluctuation into work.

Let us consider a forced ratchet system subject

Figure 1: Schematic illustration of the potential, $V(x) = V_0(x) + V_L(x)$. $V_0(x)$ is a piecewise linear and periodic potential. $V_L(x)$ is a potential due to the load, $V_L(x) = lx$. The period of the potential is $\lambda = \lambda_1 + \lambda_2$, and $\Delta = \lambda_1 - \lambda_2 (> 0)$ is a symmetry breaking amplitude. For a finite work against load, we assume the condition $l \leq A$ throughout this paper.

to an external load:

$$\frac{dx}{dt} = -\frac{\partial V_0(x)}{\partial x} + \xi(t) + F(t) - \frac{\partial V_L(x)}{\partial x}, \tag{1}$$

where x represents the state of the ratchet, $V_0(x)$ is a periodic potential, $\xi(t)$ is a thermal noise satisfying $< \xi(t)\xi(t') >= 2kT\delta(t-t')$, "$< \cdot >$" is an operator of ensemble average, $F(t)$ is an external fluctuation, $F(t+\tau) = F(t)$, $\int_0^\tau dt F(t) = 0$, and V_L is a potential due to the load, $\frac{\partial V_L}{\partial x} = l > 0$. The geometry of the potential, $V(x) = V_0(x) + V_L(x)$, is displayed in Fig. 1. The ratchet system transforms the external fluctuation into work (see, for review, Ref[10]). The model[6] Magnasco discussed is a special case of the present system, where the external load is omitted. In general, Fokker-Planck equation[13] of the system is written:

$$\begin{gathered}\frac{\partial P(x,t)}{\partial t} + \frac{\partial J(x,t)}{\partial x} = 0, \\ J = -kT\frac{\partial P(x,t)}{\partial x} + \{-\frac{\partial V_0(x)}{\partial x} + F(t) - l\}P(x,t),\end{gathered} \tag{2}$$

where $P(x,t)$ is a probability density and $J(x,t)$ is a probability current. If $F(t)$ changes slowly enough, $P(x,t)$ could be treated as quasi-static. In such situation, J can be obtained in an analytical form. For slowly changing fluctuation $F(t)$ of square wave[6] of amplitude A, we analytically obtain an average current over the period of the fluctuation,

$$J_{sqr} = \frac{1}{2}[J(A) + J(-A)]. \tag{3}$$

It is reported[6] in the operation of the ratchet that "there is a region of operating regime where the efficiency is optimized at finite temperature." The result has been interpreted that the operation of the forced thermal ratchet is helped by thermal fluctuation. This discovery has been followed and confirmed by many literatures (see the references in Ref.[10]) with various situations. We first confirm the previous report and then analyze it energetically. We can distinguish the behavior of the current J_{sqr} on the temperature into three regimes, (a) $A < \frac{Q}{\lambda_1} + l < \frac{Q}{\lambda_2} - l$, (b) $\frac{Q}{\lambda_1} + l < A < \frac{Q}{\lambda_2} - l$,

and (c) $\frac{Q}{\lambda_1} + l < \frac{Q}{\lambda_2} - l < A$; whereas the distinction is not explicitly described in the paper[6]. In regimes (a) and (b), J_{sqr} is certainly maximized at finite temperature (Fig. 2a, b). In regime (c), J_{sqr} is a monotonically decreasing function of the temperature (Fig. 2c).

We have to notice at this stage that the fluctuation-induced current J is not an energetic quantity and therefore J is only the mimic of the *energetic* efficiency. The lack of discussion of the forced ratchet system by this *real* efficiency is attributed to the lack of construction of energetics of the systems described by Langevin or equivalently by Fokker-Planck equations. Recently, an energetics of these systems was constructed by Sekimoto[11]. Therefore, we will go into the realm of the energetics of the forced thermal ratchet, and analyze the *real* efficiency.

According to the energetics[11], the input energy R per unit time from external force to the ratchet and the work W per unit time that the ratchet system extracts from the fluctuation into the work are written respectively:

$$R[F(t)] = \frac{1}{t_f - t_i}\int_{x=x(t_i)}^{x=x(t_f)} F(t)dx(t), \qquad (4)$$

$$W = \frac{1}{t_f - t_i}\int_{x=x(t_i)}^{x=x(t_f)} dV(x(t)). \qquad (5)$$

For the square wave, they yield:

$$\begin{aligned} < R_{sqr} > &= \frac{1}{2}[< R(A) > + < R(-A) >] \\ &= \frac{1}{2}A[J(A) - J(-A)], \qquad (6) \end{aligned}$$

$$< W_{sqr} > = \frac{1}{2}l[J(A) + J(-A)]. \qquad (7)$$

Therefore, we obtain the efficiency of the energy transformation η:

$$\eta = \frac{< W_{sqr} >}{< R_{sqr} >} = \frac{l[J(A) + J(-A)]}{A[J(A) - J(-A)]}. \qquad (8)$$

Figure 2: Plot of the current J_{sqr} as a function of kT/Q. The first regime (a), $A < \frac{Q}{\lambda_1} + l < \frac{Q}{\lambda_2} - l$ ($\lambda = 1.0$, $\Delta = 1.0$, $l = 0.01$, $A = 1.0$); the second regime (b), $\frac{Q}{\lambda_1} + l < A < \frac{Q}{\lambda_2} - l$ ($\lambda = 1.0$, $\Delta = 1.0$, $l = 0.01$, $A = 1.2$); and the third regime (c), $\frac{Q}{\lambda_1} + l < \frac{Q}{\lambda_2} - l < A$ ($\lambda = 1.0$, $\Delta = 0.6$, $l = 0.01$, $A = 6.0$). Regimes (a), (b) and (c) correspond to the low, moderate and high amplitude forcing in the description of Magnasco[6] respectively. In regimes (a) and (b), increasing temperature results first in a rise and then a fall in the current.

Because $\frac{J(-A)}{J(A)} < 0$, Eq. (8) is rewritten,

$$\eta = \frac{l}{A}\left\{1 - \frac{2|\frac{J(-A)}{J(A)}|}{1 + |\frac{J(-A)}{J(A)}|}\right\}. \qquad (9)$$

In the limit $|\frac{J(-A)}{J(A)}| \to 0$, the maximum efficiency of the energy transformation for given load l and force amplitude A is realized: $\eta_{max} = \frac{l}{A}$.

In Eq. (9), we can discuss the effect of thermal fluctuation on the *energetic* efficiency of the forced thermal ratchet. As demonstrated in Fig. 3, it is proved that the efficiency is a decreasing function of temperature in all the three regimes: Because, $|\frac{J(-A)}{J(A)}|$ is monotonically increasing function of the temperature as found in Fig. 4, the efficiency η is a decreasing function of the temperature. This certainly shows that the presence of thermal fluctuation *does not* help efficient energy transformation by the ratchet, which is in contrast to the previous interpretation that thermal fluctuation could facilitates the efficiency.

We can learn here that the efficiency should be discussed energetically: The condition of maximum current does not correspond to that of the maximum efficiency. The difference is attributed to the observation that the efficiency is a ratio of the extracted work W to the consumed energy R. The extracted work W is surely proportional to the current $J_{sqr} = \frac{1}{2}[J(A)+J(-A)]$ (Eq. 3). However, the consumed energy is not a constant but varies sensitively according to the condition. Therefore the efficiency η is not simply proportional to the induced current J. The important problem was left for future study whether the existence of thermal fluctuation could facilitate the efficiency of energy transformation in more general forced ratchets.

Finally, we mention that the complementarity relation[12] of the ratchet system. We found that the maximum efficiency, $\eta_{max} = 1$, can be realized if all of the the following conditions are satisfied, $A \to Q/\lambda_1 + l + 0$ and $Q \to +0$ and $T \to 0$. In this limit, the speed of energy transformation goes to zero. That is to say, this maximum efficiency of the forced ratchet is realized in quasistatic process. As we increase the velocity of this engine, the efficiency is decreased. The result also emphasizes the importance of time scales of the operation of the ratchet as Jülicher et al. pointed out[10]. Detailed analysis of the loss of the efficiency may be analyzed by

Figure 3: Plot of the efficiency η as a function of kT/Q. In each regime (a), (b) and (c), the condition is the same as in Fig.2. In all the regimes, increasing temperature results in decreasing the efficiency.

the formal theory of complementarity relation[12] between the time lapse of thermodynamic process and the irreversible heat.

References

[1] For example, Woledge, R. C., Curtin, N. A., & Homsher, E., *Energetic Aspect of Muscle Contraction* (Academic Press, New York, 1985).

[2] Feynman, R. P., Leighton, R. B., & Sands, M., *The Feynman Lectures in Physics* (Addison-Wesley, Massachussets, 1966).

[3] Büttiker, M. Transport as a consequence of state-dependent diffusion. *Z. Phys.* **B 68,** 161-167 (1987).

[4] Landauer, R. Motion out of noisy States. *J. Stat. Phys.* **53,** 233-248 (1988).

[5] Vale, R. D. & Oosawa, F. Protein motors and Maxwell's demons: Does mechanochemical transduction involve a thermal ratchet? *Adv. Biophys.* **26,** 97-134 (1990).

[6] Magnasco, M. O. Forced thermal ratchet. *Phys. Rev. Lett.* **71**, 1477-1481 (1993).

[7] Ajdari, A., Mukamel, D., Peliti, D., & Prost, J. Rectified motion induced by ac forces in periodic structures. *J. Phys. I (France)* **4**, 1551-1561 (1994).

[8] Doering, A. R., Horsthemke, W., & Riordan, J. Nonequilibrium fluctuation-induced transport. *Phys. Rev. Lett.* **72**, 2984-2987 (1994).

[9] Hondou, T., & Sawada, Y. Dynamical behaviour of a dissipative particle in a periodic potential subject to chaotic noise: Retrieval of chaotic determinism with broken parity. *Phys. Rev. Lett.* **75**, 3269-3272 (1995).

[10] Jülicher, F., Ajdari, A., & Prost, J. Modeling molecular motors. *Rev. Mod. Phys.* **69,** 1269-1281 (1997).

Figure 4: Plot of currents $J(A)$, $J(-A)$ and $\left|\frac{J(-A)}{J(A)}\right|$. The condition is the same as in the second regime (b) in Fig. 2. $|J(-A)|$ increases slower than $|J(A)|$ when kT increases from zero temperature. This difference is attributed to the symmetry breaking of the potential as illustrated in Fig. 1.

[11] Sekimoto, K. Kinetic characterization of heat bath and the energetics of thermal ratchet models. *J. Phys. Soc. Jpn.* **66,** 1234-1237 (1997).

[12] Sekimoto, K. & Sasa, S. Complementarity Relation for Irreversible Process Derived from Stochastic Energetics. *J. Phys. Soc. Jpn.* **66,** 3326-3328 (1997).

[13] Gardiner, C. W., *Handbook of Stochastic Methods 2nd ed.* (Springer-Verlag, Berlin, 1990).

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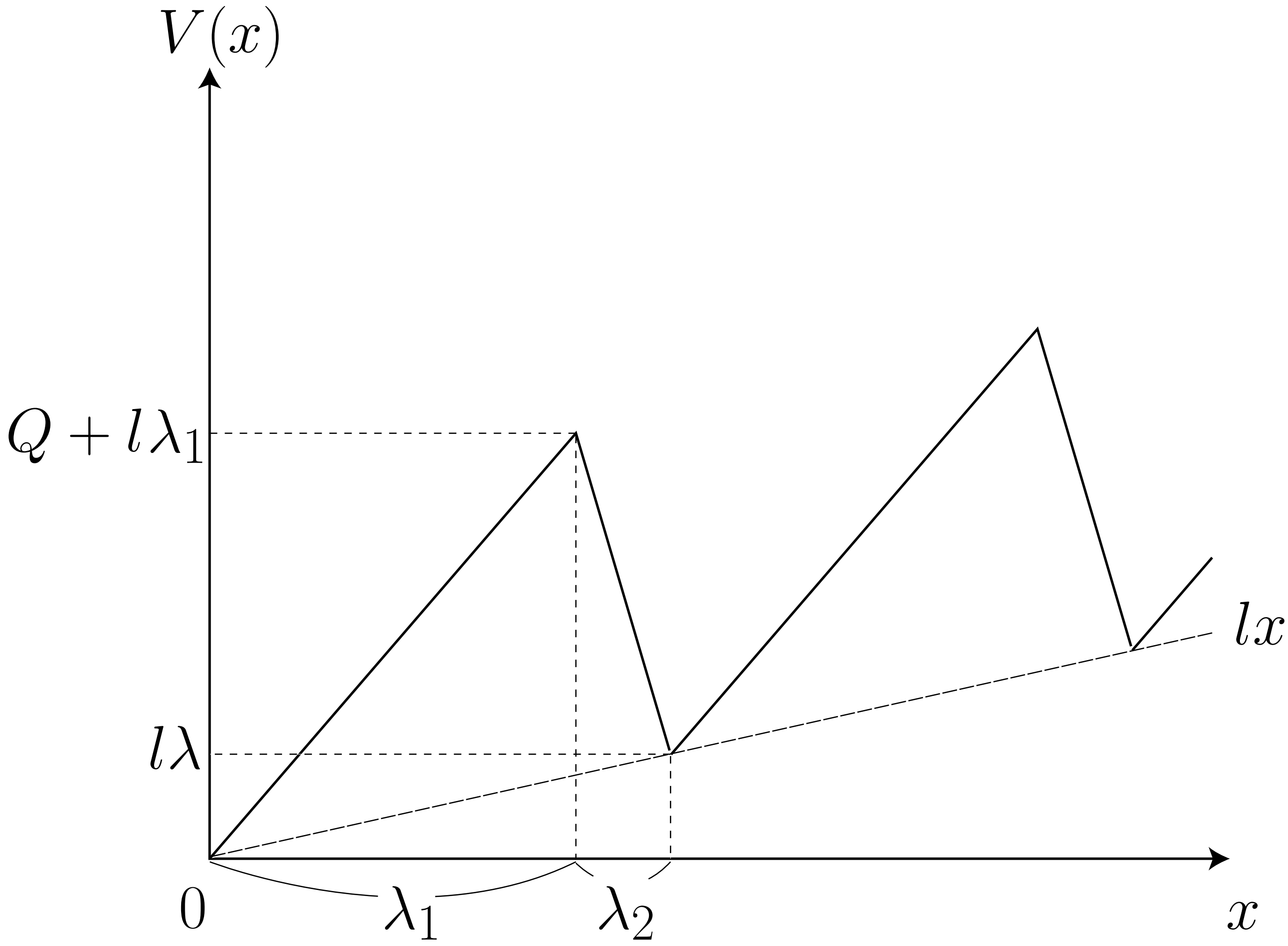
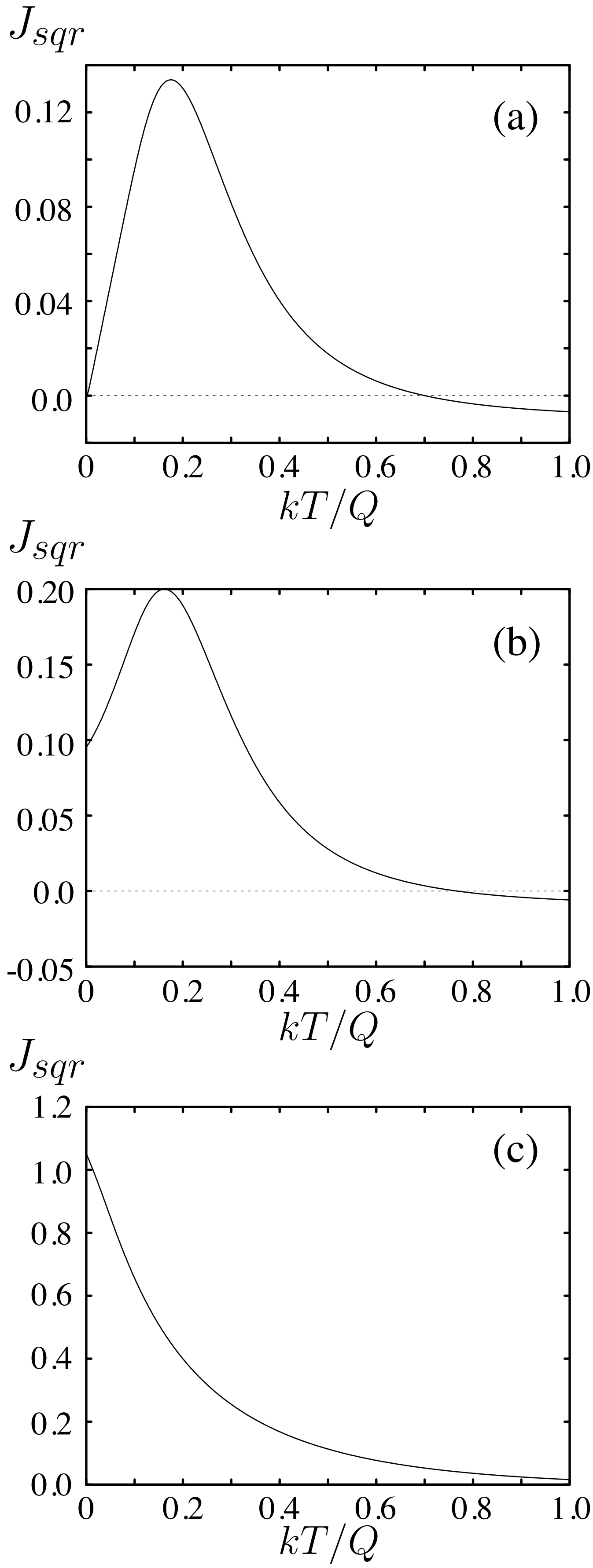
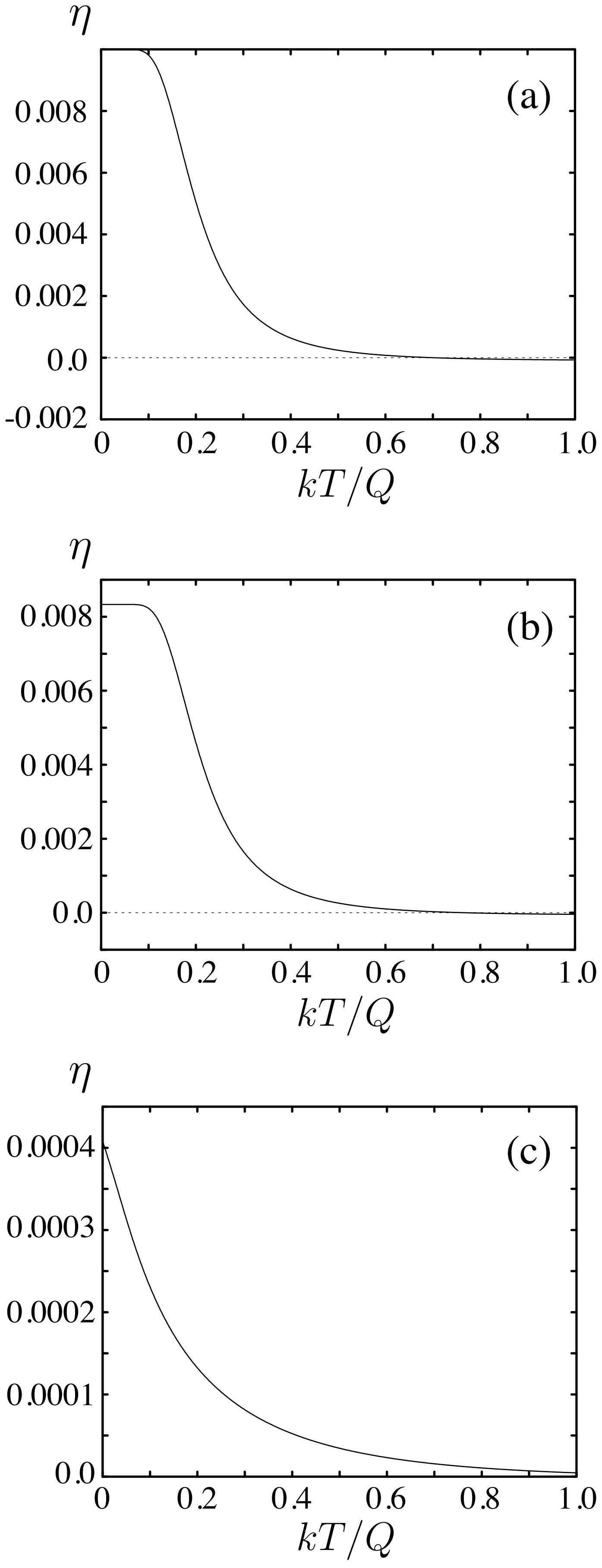
Kamegawa: FIG. 1



Kamegawa: FIG. 2



Kamegawa: FIG. 3



Kamegawa: FIG. 4